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Geraldton Senior High School

**ATMAM Mathematics Methods**

**Marking Key:** INVESTIGATION 2Random Samples and Sampling Distributions

**50 marks**

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| **Question 1.**  **a) (5 marks)** | | | |
| **Grade** | **Interpret the task and choose the mathematics** | **Apply mathematical knowledge to obtain a solution** |  |
| A | For each line of code, explains in detail the purpose of the code. Specifically for   * explains the purpose of * purpose of   Explains why the cursor needs to start on the second line for each iteration after Experiment 1 | | **5 marks** |
| B | Provides a clear explanation for each line of code including the placing of the cursor on the second line | | **4 marks** |
| C | Provides a perfunctory explanation for most lines of code.  Does not explain with and that the sample size increase by 5 with each experiment. | | **3 marks** |
| D | Provides a rudimentary explanation for less than half of the code | | **1-2 marks** |
| **Question 1.**  **b) 5 marks)** | | | |
| **Grade** | **Interpret the task and choose the mathematics** | **Apply mathematical knowledge to obtain a solution** |  |
| A | Explains the content of each list in detail. Specifically   * Clarifies why list2 has a different number of rows * Clarifies that list2 is over written with each experiment * Explains that the sample size of Experiment 5 is 25 | | **5 marks** |
| B | Provides a clear explanation for each list but does not give all the details indicated above | | **4 marks** |
| C | Provides a perfunctory explanation for each list | | **3 marks** |
| D | Does not provide a satisfactory explanation of the lists | | **1-2 marks** |

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| **Question 1.**  **c) (6 marks)** | | | |
| **Grade** |  | **Apply mathematical knowledge to obtain a solution** | **Interpret and communicates results and conclusions** |
| **A** | **5-6 marks** | Constructs accurate, clearly and fully labelled graphs that are appropriately scaled   * using a histogram / boxplot to represent the distribution of Sample Proportion Means * using simultaneous line graphs to track and compare the behaviour of   + Mean of the sample proportions   + Population Proportion   + Measured Standard Deviation   + and | From the histogram / boxplot Identifies that on after 5 experiments the distribution is not clearly symmetric.  From the line graphs indicates that   * the measured mean appears to converge on the population mean * the measured standard deviation is decreasing and closely tracking |
| **B** | **4 marks** | Graphs are accurate though not fully labelled or scaled, and has not been plotted | Observes the apparent convergence of the Mean to the Population Proportion, but unable to make comparisons between Measured Standard Deviation and |
| **C** | **3 marks** | Graphs are accurate but lacking labels of scales | Observes the apparent convergence of the Mean to the Population Proportion, but unable to make comparisons between Measured Standard Deviation and |
| **D** | **1-2 marks** | Graphs are incomplete/missing and/or incorrect | Unable to make pertinent observations regards the data or its representations |

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| **Question 2 (6 marks) Question 3 (6 marks)** | | | |
| **Grade** | **Question 2** | **Apply mathematical knowledge to obtain a solution** | **Question 3** |
| **A** | **5-6 marks** | Constructs accurate, clearly and fully labelled graphs that are appropriately scaled including a   * histogram / boxplot for Experiment 50 * simultaneous line graph tracking   + Mean of sample proportions   + Population Proportion * simultaneous line graph tracking   + Measured Standard Deviation   + and | **5-6 marks** |
| **B** | **4 marks** | Constructs accurate, clearly and mostly labelled graphs that are appropriately scaled including a   * histogram / boxplot for Experiment 50 * simultaneous line graph tracking   + Mean of sample proportions * simultaneous line graph tracking   + Measured Standard Deviation | **4 marks** |
| **C** | **3 marks** | Constructs accurate graphs that lacking clear scaling including a   * histogram / boxplot for Experiment 50 * simultaneous line graph tracking   + Mean of sample proportions * simultaneous line graph tracking   Measured Standard Deviation | **3 marks** |
| **D** | **1-2 marks** | Graphs lack accuracy / labelling / scaling / and graphs are not complete | **1-2 marks** |

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| **Question 4 (8 marks)** | | | |
| **Grade** |  | **Apply mathematical knowledge to obtain a solution** | **Interpret and communicates results and conclusions** |
| A |  | Calculates the number of data points processed in total in **Case 1** and **Case 2**.  Calculates the number of samples used for calculating Mean and S.D. for **Case 1** and **Case 2**.  By inspection, observes that **Case 1** measured mean converges more quicky on the population mean than for **Case 2**. Quantifies that observation referring to size of swing between min/max values and/or size of deviation of measured mean from the sample proportion.  By inspection, observes that **Case 1** measured SD converges more quicky on than **Case 2**. Quantifies that observation referring to size of swing between min/max values and/or size of deviation of measured S.D. from  Identifies that for **Case 2** is fixed, and quantifies for **Case 1** and Case 2. Quantifies the change in ratio for Case 1: Case 2 from Experiment 26 to Experiment 50.  For **Case 1** determines for  then compares this to  **Case 2**  and with S.D.=  **(4 marks)** | Identifies that **Case 1** & **Case 2** process the same number of data points in total.  Identifies the fact that the Mean and S.D. are calculated from a much larger number of Sample Proportions in **Case 2**. Acknowledges an apparent if superficial advantage for **Case 2** from this fact.  Concludes that the data indicates that the measured mean converges on the population mean more quickly for **Case 1** than **Case 2**.  Concludes that the data indicates that the measured S.D. converges to more quickly for **Case 1** than **Case 2**.  Concludes explicitly that and diverge to the advantage of **Case 1** with  Concludes that by the 50th experiment    For the same values of and .  Concludes that sample size has a greater effect than repeated sampling in achieving an accurate sample proportion.  Concludes that, given the relatively low sample size required for high accuracy, the quality of surveying will be the determining factor in achieving a reliable sample proportion.  **(4 marks)** |
| B |  | By inspection, observes that **Case 1** measured mean converges more quicky on the population mean than for **Case 2**. Provides some quantification to justify this.  By inspection, observes that **Case 1** measured SD converges more quicky on than **Case 2**. Provides some quantification to justify this.  Identifies that for **Case 2** is fixed, and quantifies for **Case 1** and **Case 2**. Compares the two S.D.’s  For **Case 1** determines for  then compares this to  **Case 2**  and with S.D.=  **(3 marks)** | Concludes that the data indicates that the measured mean converges on the population mean more quickly for **Case 1** than **Case 2**.  Concludes that the data indicates that the measured S.D. converges to more quickly for **Case 1** than **Case 2**.  Concludes explicitly that and diverge to the advantage of **Case 1**  Concludes that by the 50th experiment    For the same values of and .  Concludes that sample size has a greater effect than repeated sampling in achieving an accurate sample proportion. **(3 marks)** |
| C |  | By inspection, observes that **Case 1** measured mean converges more quicky on the population mean than for **Case 2**.  By inspection, observes that **Case 1** measured SD converges more quicky on than **Case 2**.  Compares the S.D.’s for **Case 1** and **Case 2**  Indicates (with some justification) that **Case 1**  If using for the former and for the latter.  **(2 marks)** | Concludes that the data indicates that the measured mean converges on the population mean more quickly for **Case 1** than **Case 2**.  Concludes that the data indicates that the measured S.D. converges to more quickly for **Case 1** than **Case 2**.  Concludes explicitly that and diverge to the advantage of **Case 1**  Concludes **informally** that by the 50th experiment    For the same values of and . **(2 marks)** |
| D |  | Concludes by inspection the measured mean/SD converges more quickly for **Case 1** than **Case 2**.  Informally suggests, without justification, that the Normal Distributions favour the sample proportion  for **Case 1** over **Case 2.**  **(1-2 mark)** | |

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| **Question 5 (14 marks)** | | | |
| **Grade** | **Interpret the task and choose the mathematics** | **Apply mathematical knowledge to obtain a solution** | **Interpret and communicates results and conclusions** |
| **A** | Identifies the need to compare the Sampling Distribution for different sample sizes, and compare these Sampling Distributions to a Normal Distribution.  From the provided tool selects “Fit to Normal” to make this comparison  From the provided tool, for maximum contrast choses sample sizes of 5 and 25 to simultaneously compare the Sampling Distribution.  Identifies the need to use a large number of repeated samples to make suitable comparisons. From the provided tool selects Nº of repeats ≥ 100 000.  Understands the need to make comparisons of samples from a variety of distributions. Uses the tool to model and compare the Sampling Distributions from **four** distributions: Normal,  Uniform, Skewed, Custom.  **5 marks** | Runs the simulation tool to obtain **four** sets of simultaneously graphed Sampling Distributions of sample size 5 and 25.  Provides clear titles and labels for each set of graphs.  Has used the tool to fit a Normal curve to each Sampling Distribution.  **4 marks** | Compares the Sampling Distribution for each of the parent distributions to the fitted Normal curve. Discussing the symmetry of Sample Distributions and relative locations of Mean and Median.  Compares the relative fit of the Sampling Distributions of sample size 5 & 25 to the Normal curve to conclude that whilst both n=5 and n=25 provide a good fit (in most cases for n=5), n=25 produces a smaller SD and that for n=25, a higher proportion of the sample means are closer to the population mean.  Explicitly concludes that the Sampling Distribution of the Sample Means is normally distributed.  Understands and explains that this applies to the Sampling Distribution of the Sample Proportions, since a Sample Proportion is a summary statistic.  Explicitly concludes that the Sampling Distribution of the Sample Proportions is normally distributed.  **5 marks** |
| **B** | Compares the Sampling Distribution for different sample sizes to a Normal Distribution.  From the provided tool selects “Fit to Normal” to make this comparison  From the provided tool, for maximum contrast choses sample sizes of 5 and 25 to simultaneously compare the Sampling Distribution.  Identifies the need to use a large number of repeated samples to make suitable comparisons. From the provided tool selects Nº of repeats ≥ 100 000.  Uses the tool to model and compare the Sampling Distributions from **three** distributions: Normal, Uniform & Skewed. **4 marks** | Runs the simulation tool to obtain **three** sets of simultaneously graphed Sampling Distributions of sample size 5 and 25.  Provides clear titles and labels for each set of graphs.  Has used the tool to fit a Normal curve to each Sampling Distribution.  **3 marks** | Compares the Sampling Distribution for each of the parent distributions to the fitted Normal curve.  Concludes that whilst both n=5 and n=25 provide a good fit (in most cases for n=5), n=25 produces a Sampling Distribution with a smaller SD. Interprets this to conclude that for n=25, a higher proportion of the sample means are closer to the population mean.  Explicitly concludes that the Sampling Distribution of the Sample Means is normally distributed and concludes that the Sampling Distribution of the Sample Proportions is normally distributed.  **4 marks** |
| **C** | Compares the Sampling Distribution to a Normal Distribution.  From the provided tool selects “Fit to Normal” to make this comparison  From the provided tool, choses just one sample sizes ≥ 20 to fit the Sampling Distribution to a Normal one.  From the provided tool uses Nº of repeats ≥10 000 but ≤100 000  Choses to model and compare the Sampling Distributions from **two** distributions: Normal and Uniform **3 marks** | Runs the simulation tool to obtain **two** sets of simultaneously graphed Sampling Distributions of one sample size ≥20.  Provides sufficient labelling of graphs.  Has used the tool to fit a Normal curve to each Sampling Distribution.  **2 marks** | Compares the Sampling Distribution for each of the parent distributions to the fitted Normal curve.  Explicitly concludes that the Sampling Distribution of the Sample Means is normally distributed and concludes, without justification, that the Sampling Distribution of the Sample Proportions is normally distributed.  **3 marks** |
| **D** | Compares the Sampling Distribution to a Normal Distribution.  From the provided tool, choses just one sample sizes < 20 observe nature of the Sampling Distribution.  From the provided tool uses Nº of repeats ≥10 000 but ≤100 000  Choses to model and compare the Sampling Distributions from **one** distributions. **1-2 marks** | Runs the simulation tool to obtain **one** set of simultaneously graphed Sampling Distributions of one sample size ≤20.  Insufficient labelling of graphs.  Has not used the tool to fit a Normal curve to each Sampling Distribution.  **1 marks** | Judges by eye that the Sampling Distribution is a reasonable fit to the Normal curve.  Explicitly concludes that the Sampling Distribution is normally distributed without awareness of Sample Means and Sample Proportions.  **1-2 marks** |